Chapter 10: Introduction to Inference
Inference

- Inference is the statistical process by which we use information collected from a sample to infer something about the population of interest.

- Two main types of inference:
  - Interval estimation (Section 10.1)
  - Tests of significance (“hypothesis testing”) (Section 10.2)
Constructing Confidence Intervals

- Activity 10, pp. 534-535
- Interpretation of 95% C.I., p. 535
  - If the sampling distribution is approximately normal, then the 68-95-99.7 rule tells us that about 95% of all p-hat values will be within two standard deviations of p (upon repeated samplings). If p-hat is within two standard deviations of p, then p is within two standard deviations of p-hat. So about 95% of the time, the confidence interval will contain the true population parameter p.
Internet Demonstration, C.I.

- http://bcs.whfreeman.com/yates/pages/bcs-main.asp?s=00020&n=99000&i=99020.01&v=category&o=&ns=0&uid=0&rau=0
Interpretation of 95% CI (This is the one you should commit to memory!)

- 95% of all confidence intervals constructed in the same manner will capture the true population parameter.
- 5% of the confidence intervals created will not capture the population parameter.
Caveat

- Complex statistical inference procedures are worthless without good data!

- When using statistical inference, we are acting as if the data are a random sample or come from a randomized experiment.
Homework

- Careful reading, pp. 535-555
Writing, 3-5 minutes:

- Explain what we did with the thumbtack problem in constructing a confidence interval. How did we do it? What is the point of constructing confidence intervals? How do we interpret our confidence interval? What does “95% confidence” mean?

- Key words:
  - Parameter, sample, statistic, repeated sampling
Example 10.2, p. 537

- See bulleted list, p. 538
- Assume we know $\sigma$.
  - In practice, this is almost never the case!
  - We use this assumption as a means for slowly introducing the ideas of statistical inference.
Example 10.2 (Figure 10.3)

Population
\[ \mu = ? \]
\[ \sigma = 100 \]

\[
\begin{align*}
\text{SRS } n &= 500 \\
\bar{x} &\pm 9 = 461 \pm 9
\end{align*}
\]

\[
\begin{align*}
\text{SRS } n &= 500 \\
\bar{x} &\pm 9 = 455 \pm 9
\end{align*}
\]

\[
\begin{align*}
\text{SRS } n &= 500 \\
\bar{x} &\pm 9 = 463 \pm 9
\end{align*}
\]

\[ \text{95\% of these intervals capture the unknown } \mu \]
Example 10.2 (Figure 10.4)
Interpretation of 95% confidence interval:

- “I am 95% confident that the percentage of men ... is between 54% and 60%.
- Ok, so what does that mean?!
  - 95% of all confidence intervals constructed in the same manner (SRS, same n) will contain the population parameter.
  - 5% of the time the CI constructed will not contain the population parameter of interest.
    - We will be wrong 5% of the time.
Exercises

- 10.1, 10.2, and 10.3, p. 542

- Know exact wording for confidence interval interpretations!
Conditions for Constructing a Confidence Interval for Estimating a Mean ($\mu$)

- The data come from an SRS from the population of interest; and
- The sampling distribution of $x$-bar is approximately normal.
  - When can we be confident that this is the case?
    - If the original (underlying) distribution is normal, then it does not matter what sample size you use.
    - If we do not know about the underlying distribution, or if we know that it is not normal, the Central Limit Theorem tells us that if $n$ is large enough, the sampling distribution will be normal.
      - $n > 25$ or $30$ guarantees it.
Step 1: Identify the population of interest and the parameter we want to draw conclusions about.

Step 2: Choose the appropriate inference procedure.
- Verify the conditions for using the selected procedure!

Step 3: Carry out the inference procedure:
- For confidence intervals: CI = estimate ± margin of error

Step 4: Interpret our results in the context of the problem.
Practice

- Exercises:
  - 10.5, p. 548
  - 10.7, p. 549
Assessing Normality

- Suppose that we obtain a simple random sample from a population whose distribution is unknown. Many of the statistical tests that we perform on small data sets (sample size less than 25-30) require that the population from which the sample is drawn be normally distributed.
  - One way we can assess whether the sample is drawn from a normally-distributed population is to draw a histogram and observe its shape.
  - What should it look like?
- What other ways can we assess whether we have drawn a sample from a normally-distributed population?
This method works well for large data sets, but the shape of a histogram drawn from a small sample of observations does not always accurately represent the shape of the population. For this reason, we need additional methods for assessing the normality of a random variable when we are looking at sample data.

- The normal probability plot is used most often to assess the normality of a population from which a sample was drawn.
Normal Probability Plots
(pp. 106-107 in your text)

- A normal probability plot shows observed data versus normal scores.
  - A normal score is the expected Z-score of the data value if the distribution of the random variable is normal. The expected Z-score of an observed value will depend upon the number of observations in the data set.
  - See Example 2.12, p. 106 for details.
- If sample data is taken from a population that is normally distributed, a normal probability plot of the actual values versus the expected Z-scores will be approximately linear.
  - In drawing the straight line, you should be influenced more by the points near the middle of the plot than by the extreme points.
From Chapter 9, Sampling Distributions

(a) \( n = 1 \)

(b) \( n = 2 \)

(c) \( n = 10 \)

(d) \( n = 25 \)
Example 10.4, p. 544
Computing exact confidence intervals for other than 95%

\[ C.I. \text{ for mean: } \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \]
Practice/HW

- Exercises, pp. 556-557:
  - 10.19, 10.20, 10.22, 10.24
Confidence Intervals with the Calculator

- Try problem 10.22 with your calculator.

\[
\text{ZInterval} \\
\text{Inpt: L1 & Stats} \\
\sigma: 8 \\
\text{List:L1} \\
\text{Freq:1} \\
\text{C-Level:.9} \\
\text{Calculate}
\]

\[
\begin{align*}
\text{ZInterval} & \quad (22.565, 28.768) \\
\bar{x} &= 25.6666666667 \\
S_x &= 8.324308839 \\
n &= 18
\end{align*}
\]
How Confidence Intervals Behave

- Problem 10.10, p. 551

- Bulleted list, pp. 549-550
  - Margin of error: $z^* \frac{\sigma}{\sqrt{n}}$
  - What happens as our confidence level increases?
  - What happens as our standard deviation changes?
  - What happens as we increase sample size, $n$?
Choosing Sample Size

- Box, p. 552

$$z^* \frac{\sigma}{\sqrt{n}} \leq m$$

- Exercise 10.12, p. 552
Reading: pp. 559-571
Exercise 10.27 (p. 564) and 10.33 (p. 569)
10.1 Quiz tomorrow
One of the most useful and common types of statistical inference.

Goal:

To assess the evidence provided by data about some claim concerning a population of interest.
Performing a Test of Significance

- Exercises 10.27, p. 564 and 10.33, p. 569

- Steps:
  - Identify the population and parameter of interest
  - State Null and Alternate Hypotheses.
  - Sketch distribution with point of interest.
  - Check conditions for performing the test.
  - Perform the significance test, including finding the appropriate p-value.
  - Draw conclusions based upon your level of significance (alpha, $\alpha$).
Terms

- Null hypothesis (p. 565)
  - No effect or no change in the population

- Alternative hypothesis
  - There is an effect or change.

- P-value (p. 567)
  - The probability that the observed outcome would take a value as extreme or more extreme than that actually observed, given that the null is true.

- Alpha ($\alpha$)
  - A set level for rejecting the null hypothesis. Compare to the p-value obtained.
Exercise 10.33, p. 569

- Part (b): “If the P-value is as small or smaller than alpha, we reject the null hypothesis, significant at level alpha.”
- Box, p. 569
Example 10.9, p. 560
Figure 10.10, p. 562

Sampling distribution of $\bar{x}$ when $\mu = 0$

$\frac{\sigma}{\sqrt{10}} = 0.316$

$\mu = 0$

$\bar{x} = 0.3$

$\bar{x} = 1.02$
Could our result have occurred by chance?

- What is the probability that we could have obtained a sample average (x-bar) of 1.02 if the population parameter were really 0?

\[
z = \frac{1.02 - 0}{\frac{1}{\sqrt{10}}} = 3.23
\]

\[p(z \geq 3.23) = ?\]
One interpretation of p-value

- The p-value is the smallest level alpha at which we would reject the null hypothesis and make our conclusion based upon the alternative hypothesis.
Practice

Problems:

- 10.28, p. 564
- 10.34, p. 569
- 10.35, p. 569
Stating Hypotheses

- One-sided test:
  - We are interested only in deviations from the null hypothesis in one direction.

- Two-sided test:
  - We just want to know if we have a difference, which could be in either direction (high or low).

- Exercises 10.29-10.32, p. 567
Step 1: Identify the population of interest and the parameter we want to draw conclusions about.
- State Null and Alternative Hypotheses.

Step 2: Choose the appropriate inference procedure.
- Verify the conditions for using the selected procedure!

Step 3: Carry out the inference procedure:
- For tests of significance: Calculate the test statistic and find the p-value.

Step 4: Interpret our results in the context of the problem.
Inference Toolbox, cont.

- **Step 2**: Choose the appropriate inference procedure.
  - Verify the conditions for using the selected procedure!
    - SRS, normal sampling distribution

- **Step 3**: Carry out the inference procedure:
  - For tests of significance: Calculate the test statistic and find the P-value.
  - P-value: describes how strong the case is against $H_0$, because it is the probability of getting an outcome as extreme or more extreme than the actually observed outcome, if the null hypothesis is true.
Step 4: Interpret our results in the context of the problem.

- Compare the p-value with a fixed value that we regard as decisive. This fixed value is called the significance level, alpha (\( \alpha \)).
- If the P-value is as small or smaller than alpha, we reject the null and have statistical significance at level \( \alpha \).
Example 10.13, p. 573

- Two-tailed test.
- Note in step 3 the doubling of probabilities to get the correct P-value for a two-tailed test.
\( H_a: \mu \neq \mu_0 \) is \( 2P(Z \geq |z|) \)
Example 10.13, cont.

\[ P = 0.2758 \]

\[ -1.09 \quad 0 \quad 1.09 \quad z \rightarrow \]
Practice Exercise

- 10.38, p. 576
- Follow the Inference Toolbox.
- After doing this by hand, let’s see how it looks on the calculator.
Homework

- Reading through p. 583
- Exercises:
  - 10.36 and 10.37, p. 570
  - 10.39, p. 576
Performing a 2-sided significance test with a C.I.

- If the null hypothesis mean falls outside of the $1 - \alpha$ Confidence Interval, we can reject the null hypothesis.
  - Example 10.17, p. 581
- Problem, 10.44, p. 582
Tests with fixed significance levels

In the old days of significance testing, we always compared our p-value to a fixed level of alpha. This is not so any more, though we still use alpha as a way to guide our decisions.

***The p-value is the smallest level alpha at which we would reject the null hypothesis and make our conclusion based upon the alternative hypothesis.
Optional practice problems for 10.2 Quiz:
- 10.81, p. 609
- 10.45, p. 583
- 10.53, p. 584

Required:
- Reading pp. 593-605 (Type I and Type II errors, power)
- Exercise 10.69, parts a-d, p. 599
Errors in significance testing

- Is it possible that we will make the wrong decision with our significance test?
Errors, cont.

Types of Error:

- **Type I**: Rejecting the null hypothesis \((H_0)\) when in fact it is true.
  - The probability of making a Type I error is exactly alpha.

- **Type II**: Failing to reject an incorrect null hypothesis.
  - A little more effort is required to calculate the probability of making a Type II error.
Power (p. 599)

- Definition: Probability of rejecting a false null hypothesis, given a specific alternate.

- A high probability of a Type II error for a particular alternative means that the test is not sensitive enough to usually detect the alternative.
  - We have low POWER in this instance.

- The power of a test against any alternative is 1 minus \( P(\text{Type II error}) \) for that alternative.
Example 10.21, p. 595

Note: $P(\text{Type I error})$—light shaded area. $P(\text{Type II error})$—dark shaded area.
Example 10.21, cont.

- If the sampling distribution from this example is narrower, what happens to the probability of making a Type II error?
  - How do we get a narrower sampling distribution?
Type II Error and Power

Exercise 10.69, parts a-d, p. 599
Increasing the power (p. 601):
- Increase alpha.
- Consider an alternative that is farther away from the null hypothesis mean.
- Increase the sample size.
- Decrease $\sigma$.

Which one(s) do you think is (are) most controllable?

http://www.intuitor.com/statistics/T1T2Errors.html
Homework

- Exercise 10.66, p. 598
- Chapter 10 test on Wednesday